Sequentially-Revealed Price Information and Consumer Choice

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Abstract

This article aims to explain an expenditure-minimizing consumer’s behavioral pattern in the situation that the prices of a good are sequentially revealed. There are some markets in which consumers usually have limited price information. A general solution and three other possible solutions are proposed and compared. The first solution or strategy turns out to be most ideal on the grounds that the average price converges to the minimum price and the standard deviation also converges to zero as the number of sellers increases to infinity. This is also supported by the simulation results. However, the most appropriate choices made by consumers depend on their risk attitudes and the number of sellers. It is because there exists the trade-off relationship between the average price and the variance. The findings through the simplified model of this study, along with further research on this topic in the future, are expected to extend to a complementary theoretical foundation of consumer choice.

Keywords: consumer choice, expenditure minimization, price expectation, risk, sequential information

1. Introduction

Price is the most essential element of a market, in which buyers and sellers interact with each other. According to [1], price can be defined as the exchange value of a good or service expressed in terms of money. Although the price of a good or service reflects its quality at times, consumers want to pay less than their willingness to pay at all times. The difference between the actual price and the willingness to pay is called consumer surplus [2]. Thus consumers usually search for the lowest price when they purchase a good or service. Even for an identical product, however, there are many sellers in the market and they may charge different prices. Further, the information about these prices may not be fully given to consumers for comparison. Such a situation is the motivation of this study.

This article analyzes a market in which the different prices of a commodity are sequentially revealed to a consumer. For example, consider a long-distance driver who wants to minimize the expenditure on gas. The consumer would try to find the lowest price gas station along the way. Similar situations were analyzed by some researchers [3-9]. They characterized the markets whose participants act under imperfect information. In most cases, a search cost was regarded
as the most important factor in determining whether to keep on searching or not. It is true, more often than not, that a consumer incurs a considerable search cost. In this article, however, a search cost is not a critical factor because it is assumed that a consumer can get price information at a very low and negligible cost. The primary concern of this article is to trace the best strategy for an expenditure-minimizing consumer.

2. Model

2.1 Basic Assumptions

First, it is assumed that there are \( J \) sellers that sell a homogeneous good in the market and the good is infinitely divisible. Second, there are \( N \) identical consumers, each of whom is risk-neutral and needs to consume a fixed quantity \( (\delta) \) of the good. Third, each seller is able to supply the good up to the maximum quantity \( (N) \) at a constant average cost, that is, \( AC_j = MC_j = c_j \) for \( j = 1, \ldots, J \). Fourth, the sellers may charge a different price \( (p_j) \), which is considered a uniform random variable on the interval \( (a, b) \). Simply it is denoted by \( p_j \sim U(a, b) \), where \( a \) is the minimum price and \( b \) is the maximum price. Furthermore, consumers know nothing but the number of sellers and the price distribution. Based on [10], it can be considered a kind of rational expectations assumption. An actual price is revealed only after the consumer has reached the seller, whereas all the actual prices are common knowledge among the sellers. Fifth, a consumer can get to the sellers in a predetermined order (i.e. \( 1, \ldots, J \)). Thus the prices are revealed sequentially. Finally, turning back to a previous seller is virtually impossible or costs much enough for the consumer to choose not to return.

2.2 Consumer's Problem

2.2.1 General Solution: A Probabilistic Approach

Let \( q_j \) be the quantity purchased by a representative consumer from \( j \)th store. Then, the consumer's total expenditure \( (TE) \) is

\[ TE = \sum_{j=1}^{J} p_j q_j. \]  

(1)

Since \( \delta \) is the quantity needed, the following constraint should hold:
\[
\delta = \sum_{j=1}^{J} q_j
\]  

(2)

If the prices were known to the consumer, simply the lowest price would be chosen. In that case, the market equilibrium would be the same as that of the classical price competition model in which a seller acts as a price taker. Thus, each seller charges \( c \) and sells an average quantity of \( N/S/J \). However, note that the price is unknown until the consumer gets to the seller. For simplicity, suppose \( J = 2 \). What is the consumer’s best strategy at the first stage where the first seller’s price \( (p_1) \) can be observed? As defined in [11], a risk neutral consumer would not buy anything from the first seller if the price is higher than the expected price from the second seller. At the first stage \((j=1)\), the consumer’s optimization problem is

\[
\begin{align*}
\text{Min. } & \quad p_1 q_1 + E[p_2 | q_2] \\
\text{s.t. } & \quad q_1 + q_2 - \delta \geq 0, \quad q_1 \geq 0, \quad q_2 \geq 0.
\end{align*}
\]

Since the objective function is linear in the quantities, the optimal solution for a risk-neutral consumer is

\[
\begin{align*}
p_1 > E[p_2] & \quad \Rightarrow q_1 = 0, \quad q_2 = \delta, \\
p_1 < E[p_2] & \quad \Rightarrow q_1 = \delta, \quad q_2 = 0, \\
p_1 = E[p_2] & \quad \Rightarrow \text{any combination such that } q_1 + q_2 - \delta (q_1, q_2 \geq 0).
\end{align*}
\]

(3)

The probability density function of \( p_2 \) is

\[
f(p_2) = \begin{cases} 
0 & (p_2 < a) \\
\frac{1}{b-a} & (a \leq p_2 \leq b), \\
0 & (p_2 > b)
\end{cases}
\]

The expected price of the second seller is \((a+b)/2\) and the probability that \( p_2 < p_1 \) is

\[
\text{prob}(p_2 < p_1) = 1 - \text{prob}(p_2 \geq p_1) = F(p_1) - \int_{a}^{p_1} f(p_2) dp_2 - \frac{p_1 - a}{b-a}.
\]

Finally, it is straightforward that the following two conditions are equivalent:

\[
p_1 > E[p_2] \iff \text{prob}(p_2 < p_1) > \frac{1}{2}.
\]

(4)

Suppose \( J > 2 \). The consumer’s strategy at the first stage depends on the probability \( \phi \) that at least one of the other sellers \((j=2, \ldots, J)\) changes a lower price than the first seller, which is

\[
\phi = 1 - \prod_{j=2}^{J} \text{prob}(p_j \geq p_1, p_1 \geq p_1, \ldots, p_j \geq p_1) = 1 - \left( \frac{b-p_1}{b-a} \right)^{J-1}.
\]
If this probability is large enough, the consumer would decide to purchase $q_i = 0$. If $J > 2$ and $a < p_1 < b$, then $\phi$ is always greater than the probability that another seller ($j \neq 1$) charges a lower price than the first seller. That is,

$$1 - \left( \frac{b-p_1}{b-a} \right)^{J-1} = \phi > \text{prob}(p_j < p_1) = \frac{p_1-a}{b-a} \text{ for } j \neq 1.$$  \hspace{1cm} (5)

Inequality (5) has an important implication, which leads to the following proposition.

Proposition (Excessive Expectation Effect): Suppose that the prices of a good are revealed sequentially and the currently revealed price is so low that each remaining seller is more likely to charge a higher price after all. An expenditure-minimizing and risk-neutral consumer does not accept the price as long as the number of remaining sellers is large enough and the consumer expects that at least one of them will charge a lower price than the current one.

The expectation appears to be excessive because the remaining sellers are more likely to charge a higher price individually. However, it is not unreasonable. The sufficiency of the number of remaining sellers can make the consumer form such an expectation. Note that the latter part of $\phi$ in (5) vanishes as $J$ increases. According to (3) and (4), it is concluded that a risk-neutral consumer chooses $q_i = 0$ if the probability $\phi$ is greater than $1/2$ or the following inequality holds for $J \geq 2$:

$$p_i > b - (b-a) \left( \frac{1}{2} \right)^{J-1} - \frac{1}{\sqrt{2}} a + \frac{\sqrt{2} - 1}{\sqrt{2}} b.$$  \hspace{1cm} (6)

The consumer faces the same problem on arrival at $j$th stage ($j = 2, \ldots, J-1$) without purchasing at the previous stages. Thus, the generalization of the decision rule is that the consumer should choose $q_i = 0$ if the following inequality holds for $J \geq 2$ and $j \leq J-1$:

$$p_j > b - (b-a) \left( \frac{1}{2} \right)^{J-j} - \frac{1}{\sqrt{2}} a + \frac{\sqrt{2} - 1}{\sqrt{2}} b - k_j a + k_j b.$$  \hspace{1cm} (6)

where $k_1 + k_2 = 1$. As $J$ goes to infinity, $k_1$ and $k_2$ converge to 0 and 1, respectively. The right side of (6) converges to $a$, the minimum price. Finally, the consumer's best strategy (Strategy 1) is summarized as follows:

$$q_j = \begin{cases} 0 & (p_j \geq k_j a + k_j b \text{ or } j > j^*) \\ \delta & \text{(otherwise)} \end{cases} \text{ for } j = 1, \ldots, J-1 \text{ and } q_J = \delta - \sum_{j=1}^{J-1} q_j.$$  \hspace{1cm} (7)

In (7), $j^*$ is the seller that satisfies $p_j \leq k_j a + k_j b$ for the first time. A consumer chooses $q_j$ until $(J-1)$th stage and, according to the basic assumptions, the quantity at the final stage is not chosen but automatically determined by the shortage of the good below the required level.
2.2.2 Risk Sharing Solution

Although a consumer chooses \( q_j \) according to (7), there still remains a risk of loosing the ex-post best price. For instance, it may happen that a consumer does not accept the first seller's price but all the remaining sellers end up charging a higher price. Therefore, some consumers may want to purchase a certain amount of the good at the current stage to avoid the risk, although there is a real possibility of finding a lower price later. This strategy enables the consumer to reduce the risk but it does not at no cost. Suppose again \( J=2 \). According to Strategy 1, the consumer would not purchase anything from the first seller if

\[
p_1 > E(p_2) \text{ or } \text{prob}(p_2 < p_1) > \frac{1}{2}.
\]

However, if a consumer would purchase some of the good even at a higher price than \( E(p_2) \), then the next question is how much to purchase. A possible strategy for this concern is to make \( 1 - F(p_1) \) allotted as a share to the quantity purchased at \( j \)th stage. In general, the risk sharing solution (Strategy 2) can be described as

\[
q_i - \delta (1-F(p_i)), \ldots, q_{J-1} = \left( \delta - \sum_{j=1}^{J-2} q_j \right) (1-F(p_{J-1})), q_J = \delta - \sum_{j=1}^{J-1} q_J.
\]

(8)

2.2.3 Reservation Price Solutions

A reservation price can be defined as the predetermined price level at or below which a consumer is willing to buy the good [12]. Let \( r \) be the reservation price. When \( J=2 \), for example, Strategy 1 in (7) implies that the consumer's reservation price at the first stage \((j=1)\) is equal to \((a+b)/2\), it is the same as \( E(p_2) \). Let \( j^* \) be the seller whose price is equal to or lower than the reservation price for the first time. Then, a possible strategy (Strategy 3) is

\[
q_j = \begin{cases} 0 & (p_j > r \text{ or } j > j^*) \text{ for } j=1, \ldots, J-1 \text{ and } q_J = \delta - \sum_{j=1}^{J-1} q_j. \\
\delta & \text{(otherwise)}
\end{cases}
\]

(9)

For (9) to be a complete strategy, the reservation price should be predetermined. It may differ with the consumer's risk attitude. Generally speaking, however, a reservation price does not depend solely on the risk attitude. Rather, it is affected more significantly by the extent of wish and the budget as well. If a good is not affordable, then the consumer will search for another substitute. However, these factors are not considered critical in this model because it is assumed that consumers need to purchase a fixed quantity of the good in any way and there is no limit to the budget despite the expenditure-minimizing objective. Suppose again \( J=2 \) for
simplicity. A risk-neutral consumer compares $p_j$ and $E(p_j)$ at the first stage, where the latter acts as a reservation price. Since $p_j$ is an independent and identically distributed uniform random variable on the interval $(a, b)$, the consumer's expectation of the price will be the same as $(a+b)/2$ at every stage. Even when $J > 2$, the consumer's reservation price may be the same as the expected price at each stage. That is,

$$ r = E(p_j) = \frac{a+b}{2} \text{ for } j = 1, \ldots, J-1. \tag{10} $$

It is obvious that $E(p_J) = (a+b)/2$. A reservation price does not matter at the final stage. Both (9) and (10) now complete Strategy 3.

Finally, it is interesting to think about the combination of risk sharing and reservation price, namely, the risk sharing solution with a reservation price. A possible strategy (Strategy 4) is

$$ q_j = \begin{cases} 
0 & (p_j > r) \\
\delta \times \frac{F(r) - F(p_j)}{\bar{F}(r)} & (p_j \leq r, j = 1) \\
\delta \times \left( 1 - \sum_{i=1}^{j-1} q_i \right) \times \frac{F(r) - F(p_j)}{\bar{F}(r)} & (p_j \leq r, 2 \leq j \leq J-1), \text{ where } r = \frac{a+b}{2}. \\
\delta \times \left( \sum_{i=1}^{J-1} q_i \right) & (j - J)
\end{cases} \tag{11} $$

2.3 Seller's Problem: A Brief Consideration

Recall the long-distance driver's problem. The following diagram illustrates the situation that a driver has a total of $J$ gas stations to purchase a certain amount of gas on the way from $S$ to $T$. As stated above, the consumer's objective is to minimize the total expenditure on gas.

![Fig. 1] long-distance driver's problem

The $J$ sellers compete with each other in terms of price. If a seller charges the maximum price that a typical consumer is willing to pay, it will be able to serve all the consumers who adopt Strategy 1 and visit the seller at their first stages. The first seller's price ($p_i^*$) is

$$ p_i^* = \max \left\{ \frac{1}{\sqrt{2}} a + \frac{-\sqrt{2} - 1}{\sqrt{2}} b, c_i \right\}. $$
where \( c_1 \) is the first seller's marginal cost. Since \( a \) is the minimum value of the price population, it is reasonable to assume \( c_1 \leq a \). Therefore, \( p_1^* \) is given by \( k_1(a + k_2b) \). Even though the other sellers charge a lower price than \( p_1^* \), it is meaningless without price advertising. Thus, the first seller's priority over the following sellers gives them an incentive to advertise their lower prices. Some sellers would build up a huge signboard with price information so that consumers may notice it from miles away. Suppose that the first seller charges \( p_1^* \) and \( j \)th seller also charges the same price with advertising. A consumer with Strategy 1 would be indifferent between the two sellers unless a lower price is observed in the vicinity. Some would purchase from the first seller and others from \( j \)th seller.

Suppose the situation that another seller charges a lower price than \( p_1^* \) with price advertising. Then, the seller would finally attract all the consumers with Strategy 1, which, in turn, makes the other sellers quote the same price as long as their marginal costs are covered by the price. If the whole market becomes extremely competitive, a final equilibrium will be attained when all the surviving sellers charge the lowest common marginal cost, namely, \( c \). In that case, \( p_j \sim U(a, b) \), one of the most fundamental assumptions of the model, is broken. However, such a situation is unlikely to happen. There are many reasons: obstacles to price advertising, different marginal costs, incomplete information, dispersion of sellers, and so on. Furthermore, consumers may choose different strategies according to their individual characteristics. Consumers’ choices are sometimes inconsistent and unpredictable. For instance, consider \( J = 3 \). If \( p_1 < p_2 < p_3 \), then the sellers have no incentive to disclose its price on a noticeable signboard. Nonetheless, some consumers actually purchase some from the second or third seller according to their strategies. Strategies 2 and 4 of this model give such solutions. Concerning the other obstacles to price advertising, note that costly advertising would cause an increase in the price. In addition, the credibility of price advertising matters. Some sellers may try to attract more consumers by misleading information on their advertisements. Then, consumers will not rely on sellers’ price advertising any longer. Further, price advertising may be effective only in the vicinity of a small area. If sellers are scattered over a wide area, they will have a little incentive to advertise their prices. Therefore, the basic assumption of the price distribution is justified by these various situations.

3. Simulation

The analysis of the model presents four different strategies that an expenditure-minimizing consumer can adopt. Consumers are basically assumed to be risk-neutral, but this assumption
is relaxed in some situations. Not all consumers are risk-neutral in reality. This is the reason that four different strategies are considered in the model. Since the strategies have their own grounds for the conceptual validity, it is necessary for their actual evaluations to compare the performances by running the model based on the different strategies.

First, [Table 1] shows the simulation results of the four strategies when the parameters are given by $a = 3.2$, $b = 4.0$, and $\delta = 15$. With this parameter setting for simulation, the average purchasing prices and their standard deviations are approximated by 1,000 iterations using a Monte Carlo method.

**Table 1** simulation results ($a = 3.2$, $b = 4.0$, and $\delta = 15$)

<table>
<thead>
<tr>
<th>Average Price</th>
<th>J=2</th>
<th>J=4</th>
<th>J=10</th>
<th>J=20</th>
<th>J=1000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strategy 1</td>
<td>3.5048 (0.2080)</td>
<td>3.4072 (0.1848)</td>
<td>3.3313 (0.1623)</td>
<td>3.2889 (0.1443)</td>
<td>3.2059 (0.0359)</td>
</tr>
<tr>
<td>Strategy 2</td>
<td>3.5376 (0.1868)</td>
<td>3.4785 (0.1371)</td>
<td>3.4702 (0.1205)</td>
<td>3.4659 (0.1200)</td>
<td>3.4705 (0.1227)</td>
</tr>
<tr>
<td>Strategy 3</td>
<td>3.5048 (0.2080)</td>
<td>3.4244 (0.1502)</td>
<td>3.4037 (0.1169)</td>
<td>3.3995 (0.11376)</td>
<td>3.4037 (0.1195)</td>
</tr>
<tr>
<td>Strategy 4</td>
<td>3.5373 (0.2161)</td>
<td>3.4405 (0.1728)</td>
<td>3.3563 (0.0904)</td>
<td>3.3362 (0.0627)</td>
<td>3.3337 (0.0607)</td>
</tr>
</tbody>
</table>

Note: standard deviations in parentheses.

**Strategy 1** gives the lowest average price for any size of $J$, except that it ties with Strategy 3 for $J=2$. Furthermore, the average price of Strategy 1 converges to the lowest price ($\nu$) and its standard deviation converges to zero as $J$ goes to infinity. Up to some $J$, the standard deviation of Strategy 1 is greater than those of the other strategies, making a trade-off between the average price and the risk. The standard deviation of Strategy 1 has the largest value among the four strategies for $J=20$, while it has the smallest value for $J=1000$. The number of sellers is the most critical factor that influences the performances of the strategies.

It is noteworthy that Strategy 2 has a smaller variance than Strategy 1 when the number of sellers is not large. Note that Strategy 2 is a risk sharing solution. Strategy 4, another risk sharing solution, has the largest variance for $J=2$ but the variance gradually diminishes until it becomes the smallest one for $J=10$.

The stylized facts of the strategies need to be further validated by the simulation based on a
different parameter setting. Table 2 presents the simulation results when the parameters are
given by \(a = 10\), \(b = 20\), and \(\delta = 50\). These parameters are all increased compared to the values
of the previous parameter setting. The latter table makes the differences among the strategies
more noticeable but shows the same patterns of differences.

(Table 2) Simulation results \((a = 10\), \(b = 20\), and \(\delta = 50\))

<table>
<thead>
<tr>
<th>Average Price</th>
<th>(J=2)</th>
<th>(J=4)</th>
<th>(J=10)</th>
<th>(J=20)</th>
<th>(J=1000)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strategy 1</td>
<td>13.7321</td>
<td>12.6258</td>
<td>11.6340</td>
<td>11.0702</td>
<td>10.0710</td>
</tr>
<tr>
<td></td>
<td>(2.5982)</td>
<td>(2.3880)</td>
<td>(2.0685)</td>
<td>(1.7435)</td>
<td>(0.3826)</td>
</tr>
<tr>
<td></td>
<td>(2.3154)</td>
<td>(1.7204)</td>
<td>(1.4762)</td>
<td>(1.5279)</td>
<td>(1.4712)</td>
</tr>
<tr>
<td></td>
<td>(2.5982)</td>
<td>(1.8842)</td>
<td>(1.4496)</td>
<td>(1.4437)</td>
<td>(1.4441)</td>
</tr>
<tr>
<td></td>
<td>(2.6772)</td>
<td>(2.2563)</td>
<td>(1.0713)</td>
<td>(0.7761)</td>
<td>(0.7601)</td>
</tr>
</tbody>
</table>

Note: standard deviations in parentheses.

Consequently, it is argued that the number of sellers is very critical in a consumer’s decision
making of how much to purchase at the currently revealed price, while other parameters such
as \(a\), \(b\), and \(\delta\) are not. Further, a consumer’s risk attitude matters in choosing an appropriate
strategy. It is because there exists a trade-off relationship between the average price and the
variance, which are two criteria for choosing a strategy. When \(J = 10\), for instance, Strategy 4
looks desirable in that it gives a very low average price with the least standard deviation. In
reality, such a strategy can be observed more often than not. Suppose a storabile good whose
price is changeable. When the price appears to be lower than ever, consumers tend to purchase
more than usually needed. This is the same behavioral pattern as in Strategy 4. Based on the
simulation results, it is obvious that the actual performance of Strategy 4 may be better than
that of Strategy 1 by chance. In other words, there is a possibility that the actual total
expenditure of Strategy 4 is smaller than that of Strategy 1.

4. Conclusions

This study focuses on an expenditure-minimizing consumer’s behavior in the situation that
the prices of a good are sequentially revealed. Owing to the advanced information and
communication technology as well as the emphasis on consumer protection, consumers are given more detailed price information than ever. Nonetheless, there are some particular markets (e.g., traditional markets) in which consumers usually have little information about the prices. A probabilistic optimal solution and three other possible solutions are proposed and compared. Although the first strategy seems ideal in that the average price converges to the minimum price and the variance to zero as the number of sellers increases, it may not be adopted unless the number of sellers is large enough. It is because of the trade-off relationship between the average price and the variance. Thus, a consumer's risk attitude plays an important role in choosing the most appropriate strategy for some situation.

Although only a brief consideration is given to a seller's problem, the discussion leads to the grounds for the price distribution assumed in the model. The findings through the simplified model of this study are meaningful enough to extend to a complementary theoretical foundation of consumer choice.

References