Buyer’s Pricing and Lot-sizing Policy with Price Dependent Demand under Day terms Supplier Credit in a Two-Stage Supply Chain

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Abstract

As an effective means of price discrimination, some suppliers offer credit periods to the buyers in order to stimulate the demand for the product they produce. The availability of the credit period from the supplier enables the buyer to choose his selling price from a wider range of price option. Since the buyer's lot-size is affected by the customer's demand rate of the product, the problems of determining the selling price and the lot-size are interdependent and must be solved simultaneously. In this regard, this paper deals with the problem of determining the buyer’s optimal selling price and lot-size simultaneously when the supplier permits delay in payments for an order of a product. The positive effects of credit period on the product demand can be integrated into the inventory model through the consideration of retailing situations, where the demand rate is a function of the selling price the buyer sets for the product. Thus, it is also assumed that the customer’s demand rate is represented by a downward sloping linear function of the selling price. Investigation of the properties of an optimal solution allows us to develop an algorithm whose validity is illustrated using an example problem.

Keywords : Credit Period; Supply Chain; Pricing; Lot size; Linear demand function

1. Introduction

In this paper, we consider a two-stage supply chain which consists of the supplier, the buyer and his customer. An effective supply chain requires a cooperative relationship between the supplier(vendor) and buyer(distributor). The cooperation include the sharing of information, resources and profit or cost saving. One of the most common strategies is to set up a pricing policy to attract both the supplier and the buyer. Other strategies may include better credit terms and other strategic partnership advantages. In today, it is more and more common to see that the buyers are allowed some grace period before settle the account with the supplier. In this regard, a number of research papers appeared which deal with the inventory model under trade credit. Chung[1], Goyal[2] and Teng et al.[3] analyzed the mathematical model for

Received (September 21, 2017), Review Result (October 11, 2017)
Accepted (October 18, 2017), Published (December 31, 2017)
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ISSN: 2383-5281 AJMAHS
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obtaining an economic order quantity when the supplier permits a fixed delay in payments. Kreng and Tan[4] evaluated the inventory model under two levels of trade credit policy depending on the order quantity. Mahata and Goswami[5] also extended the inventory model to the case of deteriorating products under trade credit.

However, all of the research mentioned above held the assumption that the customer’s demand is a known constant, which consequently disregards effects of trade credit on the quantity demanded. According to Mehta[6], a major reason for the supplier to offer a credit period to the buyers is to stimulate the demand for the product that he produces, and the supplier usually expects that the increased sales volume can compensate the capital losses incurred during the credit period. Also, the buyers who are allowed a period to pay back for the products bought without paying any interest, can earn interest on the sales of the inventory depending on the length of that payments period. The positive effects of credit period on the product can be integrated into the EOQ model through the consideration of retailing situations where the customer’s demand rate is a function of the selling price the buyer sets of the product. The availability of the credit period from the supplier enables the buyer to choose the selling price from a wide range of the option. Since the buyer’s lot-size is affected by the demand rate of the product, the problems of determining the buyer’s selling price and the lot-size are interdependent and must be solved simultaneously (we will call the RPLS proble).

In consideration of the RPLS problem, there were numerous research papers in the literature. Abad[7][8] dealt with the RPLS problem assuming that the supplier offers quantity discounts. They considered the demand for the product as two common decreasing functions of price: (1) the constant price elasticity function, and (2) the linear demand function. Recently, Chang et al.[9], Dye and Ouyang[10], Ouyang et al.[11], Shinn[12], and Teng et al.[13] introduced the RPLS problem under trade credit when the customer’s demand rate is represented by a constant price elasticity function of selling price. Also, Tsao and Sheen[14] extended the RPLS problem to the case of deteriorating products under trade credit. Also, in the case of the linear demand function, we can find some researches but it has not been studied relatively well. Desheng[15] examined the effect of the downward slopping linear demand function on the bargaining equilibrium behavior in supply chain. Also, Avinadav et al.[16], Chen and Chang[17], and Shi et al.[18] evaluated the RPLS problem without trade credit assuming that the demand rate is represented by a linear function of selling price.

In view of this, this paper deals with the RPLS problem when the supplier offers a certain credit period and the demand of the product is represented by a linear function of selling
price. In the next section, we formulate a relevant mathematical model. The properties of an optimal solution are discussed and a solution algorithm is given in Section 3. A numerical example is provided in Section 4, which is followed by concluding remarks.

2. Model Development

2.1 Assumptions and notations

The model presented is considered a two-stage supply chain which consists of the supplier, the buyer and his customer. We evaluate the RPLS problem for the buyer’s point of view. In deriving the mathematical model the following assumptions and notations are used:

Assumptions:

(1) Replenishments are instantaneous with a known and constant lead time.
(2) The customer’s demand rate is represented by a linear function of the buyer’s selling price.
(3) No shortages are allowed.
(4) The inventory system involves only one item.
(5) The supplier proposes a certain credit period and sales revenue generated during the credit period is deposited in an interest bearing account with rate $I$. At the end of the period, the credit is settled and the buyer starts paying the capital opportunity cost for the items in stock with rate $R(R \geq I)$.

Notations:

$C =$ unit purchase cost.
$S =$ ordering cost.
$t_c =$ credit period set by the supplier.
$H =$ inventory holding cost, excluding the capital opportunity cost.
$R =$ capital opportunity cost (as a percentage).
$I =$ earned interest rate (as a percentage).
$Q =$ lot-size.
$T =$ replenishment cycle time.
$D =$ the customer’s annual demand rate, as a function of the buyer’s selling price, $D = a - bP$

where $a$ and $b$ are positive constants.

$P =$ the buyer’s selling price, $P < a/b$. 
2.2 Proposed model development

The buyer’s objective is to maximize his annual net profit $\Pi(P,Q)$ from the sales of the products. The annual net profit consists of the following five elements as stated by Goyal[2].

(1) Annual sales revenue, $A_R$

$$A_R = PD$$

(2) Annual purchasing cost, $A_P$

$$A_P = CD$$

(3) Annual ordering cost, $A_O$

$$A_O = \frac{SD}{Q}$$

(4) Annual inventory holding cost, $A_H$

$$A_H = \frac{HQ}{2}$$

(5) Annual capital opportunity cost, $A_C$

(i) Case $1(D_t \leq Q)$: (see Fig. 1(a)) As products are sold, the sales revenue is used to earn interest with annual rate $I$ during the credit period $t_c$. And the average number of products in stock earning interest during time $(0,t_c)$ is $D_t/2$ and the interest earned for order becomes $(D_t/2)\cdot(t_cCI)$. When the credit is settled, the products still in stock have to be financed with annual rate $R$. Since the average number of products during time $(t_c,Q/D)$ becomes $D(Q/D-t_c)/2$, the interest payable per order can be expressed as $(D(Q/D-t_c)/2)\cdot(Q/D-t_c)\cdot CR$. Therefore,

$$A_C = \frac{D(Q/D-t_c)/2}{Q/D} \cdot (Q/D-t_c)CR - \frac{(D_t/2)t_cCI}{Q/D} = \frac{C(R-I)D^2t_c^2}{2Q} + CRQ - CRDt_c.$$ 

[Fig. 1] Credit period($t_c$) vs. Replenishment cycle time($Q/D$)
(ii) Case 2 \( (D_t > Q) \): (see Fig. 1(b)) For the case of \( D_t > Q \), all the sales revenue is used to earn interest with annual rate \( I \) during the credit period \( t_c \). The average number of products in stock earning interest during time \( (0, Q/D) \) and \( (Q/D, t) \) become \( Q/2 \) and \( Q \), respectively. Therefore,

\[
A_C = \frac{(Q/2)(Q/D)CI+ Q(t_c-Q/D)CI}{Q/D} = \frac{CIQ}{2} - CIDt_c.
\]

The annual net profit \( \Pi(P,Q) \) can be expressed as

\[
\Pi(P,Q) = A_R - A_P - A_O - A_H - A_C.
\]

Depending on the relative size of \( D_t \) to \( Q \), \( \Pi(P,Q) \) has two different expression as follows:

(1) Case 1 \( (D_t \leq Q) \)

\[
\Pi_1(P,Q) = PD - CD - \frac{SD}{Q} \left( HQ \frac{C(R-I)D^2t^2_c}{2Q} + \frac{CRO}{2} - CRDt_c \right),
\]

(2) Case 2 \( (D_t > Q) \)

\[
\Pi_2(P,Q) = PD - CD - \frac{SD}{Q} \left( H \frac{C}{Q} - CIDt_c \right).
\]

3. Determination of Buyer’s Pricing and Lot sizing Policy

The problem is to find an optimal selling price \( P^* \) and an optimal lot-size \( Q^* \) which maximizes \( \Pi(P,Q) \). For the first and second order conditions with respect to \( Q \), we have

\[
\frac{\partial \Pi_1(P,Q)}{\partial Q} = \frac{2SD + C(R-I)D^2t^2_c}{2Q^2} - \frac{H + CR}{2},
\]

(3)

\[
\frac{\partial \Pi_2(P,Q)}{\partial Q} = -\frac{SD}{Q^2} \left( H + CI \right),
\]

(4)

\[
\frac{\partial^2 \Pi_1(P,Q)}{\partial Q^2} = -\frac{2SD + C(R-I)D^2t^2_c}{Q^3},
\]

(5)

\[
\frac{\partial^2 \Pi_2(P,Q)}{\partial Q^2} = -\frac{2SD}{Q^3}.
\]

(6)
For a fixed $P$, $\Pi(P,Q)$ is a concave function of $Q$, and there exist a unique value $Q_i$, which maximizes $\Pi_i(P,Q)$ as follows:

$$Q_1 = \sqrt{\frac{2SD}{H_1}}, \text{ where } S_i = S + \frac{C(R-I)D^2}{2} \text{ and } H_i = H + CR,$$  \hspace{1cm} (7)

$$Q_2 = \sqrt{\frac{2SD}{H_2}}, \text{ where } H_2 = H + CI.$$  \hspace{1cm} (8)

Note that since the demand rate $D$ is a function of $P$, each $Q_i$ can be represented by a real valued function of $P$; that is $Q_i = Q_i(P)$.

Now, from equations (7) and (8), we have the following useful property for a fixed $P$.

**Property 1.** $Q_i(P) \geq Dt_c$ if and only if $Q_i(P) \geq Dt_c$. If $Q_i(P) \geq Dt_c$, then $\Pi_i(P,Q)$ is increasing in $Q$ over $Q < Dt_c$. If $Q_i(P) < Dt_c$, then $\Pi_i(P,Q)$ is decreasing in $Q$ over $Q \geq Dt_c$.

**Proof.** From equation (7), $Q_i(P) \geq Dt_c$ can be rewritten as

$$\sqrt{\frac{2SD+C(R-I)D^2}{H+CR}} \geq Dt_c.$$  \hspace{1cm} (9)

Squaring both side of equation (9) and rearranging,

$$2SD+C(R-I)D^2 \geq (H+CR)D^2t_c^2,$$  \hspace{1cm} (10)

$$\sqrt{\frac{2SD}{H+CI}} \geq Dt_c.$$  \hspace{1cm} (11)

equation (11) implies that $Q_1(P) \geq Dt_c$ and $Q_2(P)$ is a maximum point of $\Pi_i(P,Q)$, which is a concave function. Thus $\Pi_2(P,Q)$ is increasing in $Q$ over $Q < Dt_c$. Also, from equations (9) and (11), $Q_2(P) < Dt_c$ implies that $Q_1(P) < Dt_c$. Likewise, $Q_1(P)$ is a maximum point of $\Pi_1(P,Q)$ and so $\Pi_1(P,Q)$ is decreasing in $Q$ over $Q \geq Dt_c$.  

Q.E.D.

Property 1 states that for a fixed $P$, the optimal lot-size $Q^*(P)$ which maximizes $\Pi(P,Q)$ is known to be either $Q_1(P)$ or $Q_2(P)$ because the annual net profit function is continuous at $Q = Dt_c$. Moreover, if $Q_1(P) \geq Dt_c$, then $Q_1(P)$ becomes $Q^*(P)$. And if $Q_1(P) < Dt_c$, then $Q_2(P) < Dt_c$ and $Q_2(P)$ becomes $Q^*(P)$.

Now, let us consider $Q_i(P) \geq Dt_c$. Since the demand rate $D$ is also a function of $P$, the inequality can be rewritten as
Rearranging equation (12),

\[ P \geq \frac{a}{b} \frac{2S}{b(H+CI)t_c^2}. \]  

Let

\[ P_0 = \frac{a}{b} \frac{2S}{b(H+CI)t_c^2}. \]  

It is self evident that for any \( P \geq P_0 \), the inequality \( Q_i(P) \geq D_{t_c} \) holds. So, we conclude that \( Q_i(P) \) determined by \( P \) value which satisfies the inequality (13) becomes an optimal lot size \( Q'(P) \). Similarly, \( Q_2(P) \) becomes an optimal lot size \( Q'(P) \) only if \( P < P_0 \). Consequently, substituting \( Q \) with \( Q_i(P) \) in \( \Pi_i(P,Q) \), we have a problem of maximizing \( \Pi_i(P,Q_i(P)) \) which is a single variable function. With \( \Pi_i^0(P) = \Pi_i(P,Q_i(P)) \) for \( P \geq P_0 \) and \( \Pi_2^0(P) = \Pi_2(P,Q_2(P)) \) for \( P < P_0 \), the following single variable objective function is obtained.

\[
\Pi_1^0(P) = D(P - C(1 - R_{t_c})) - \sqrt{2S_i DH_i} \quad \text{for} \quad P \geq P_0, \\
\Pi_2^0(P) = D(P - C(1 - R_{t_c})) - \sqrt{2SDH_2} \quad \text{for} \quad P < P_0.
\]  

Therefore, an optimal solution \((P', Q')\) which maximizes \( \Pi(P,Q) \) is found by searching over \( \Pi_1^0(P) \). Considering the characteristics of \( \Pi_1^0(P) \), we can find the following property for the shape of \( \Pi_1^0(P) \).

**Property 2.** \( \Pi_1^0(P) \) is a concave-convex-concave function of \( P \) and \( \Pi_2^0(P) \) is a concave-convex function of \( P \).

**Proof.** For Case 1, the shape of \( \Pi_1^0(P) \) can be studied by examining its first and second derivative with respect to \( P \).

\[
\Pi_1^0(P)' = \frac{d\Pi_1^0(P)}{dP} = D + D'(P - C(1 - R_{t_c})) - D'(S + C(R - I)D_{t_c}^2) \sqrt{\frac{H_i}{2S_i D'}}. \\
\Pi_1^0(P)'' = \frac{d^2\Pi_1^0(P)}{dP^2} = 2D' + \frac{2S^2}{2S_i D'} \sqrt{\frac{H_i}{2S_i D'}}. \\
\]
Then, Equation (18) can be set to zero so that

$$\Pi_i^0(P)'' = 0. \quad (19)$$

Substituting $D$ with $a - bP$ in equation (18), we have the following quadratic equation of $P$.

$$f(P) = K\delta^2 P^2 - 2(Ka + S)bP + a(Ka + 2S) - \left( \frac{1}{4} b^2 S^4 H_1^2 \right)^{1/3}, \quad K = C_i^2 (R - I). \quad (20)$$

Because $K\delta^2 > 0$ and the discriminant of $f(P)$ is positive, $f(P)$ have two real roots with $\hat{P}_{11}$ and $\hat{P}_{12}$. By the quadratic formula, we have the following two roots;

$$\hat{P}_{11} = \frac{2(Ka + S)b - \sqrt{4b^2(S^2 + K(b^2 S^4 H_1^2)/4)^{1/3}}}{2K\delta^2}, \quad (21)$$

$$\hat{P}_{12} = \frac{2(Ka + S)b + \sqrt{4b^2(S^2 + K(b^2 S^4 H_1^2)/4)^{1/3}}}{2K\delta^2}. \quad (22)$$

Therefore,

$$f(P) > 0 \text{ for } P < \hat{P}_{11}, \quad (23)$$

$$f(P) < 0 \text{ for } \hat{P}_{11} < P < \hat{P}_{12}, \quad (24)$$

$$f(P) > 0 \text{ for } \hat{P}_{12} < P. \quad (25)$$

Note that $f(P) > 0$ implies $\Pi_i^0(P)'' < 0$, which means the concave function and $f(P) < 0$ implies $\Pi_i^0(P)'' > 0$, which means the convex function. So, $\Pi_i^0(P)$ is a concave-convex-concave function of $P$.

For Case 2, the shape of $\Pi_2^0(P)$ can be studied by examining its first and second derivative with respect to $P$.

$$\Pi_2^0(P)' = \frac{d\Pi_2^0(P)}{dP} = D + D'(P - C(1 - I_{r_i})) - \frac{\delta^2}{2D^2} \sqrt{\frac{H_2SD}{2}}, \quad (26)$$

$$\Pi_2^0(P)'' = \frac{d^2\Pi_2^0(P)}{dP^2} = 2D + \frac{\delta^2}{2D^2} \sqrt{\frac{H_2SD}{2}}. \quad (27)$$

Let $\hat{P}_2$ be the solution of

$$\Pi_2^0(P)'' = 0 \quad (28)$$
and we have
\[
\hat{P}_2 = \frac{a}{b} - \frac{1}{b} \left( \frac{b}{4 \sqrt{2}} \sqrt{H_2S} \right)^2 .
\]  
(29)

Then,
\[
\Pi_2(P)'' < 0 \text{ for } P < \hat{P}_2, \quad \text{(30)}
\]
\[
\Pi_2(P)'' > 0 \text{ for } \hat{P}_2 < P. \quad \text{(31)}
\]

Therefore, we conclude that \(\Pi_2(P)\) is a concave-convex function of \(P\). Q.E.D.

Now, to find the extreme points of \(\Pi_2(P)\), let us consider the first order condition for \(\Pi_2(P)\) with respect to \(P\) as follows;
\[
\Pi_2(P)' = \frac{d\Pi_2(P)}{dP} = 0, \quad \text{(32)}
\]
\[
\Pi_2(P)' = \frac{d\Pi_0(P)}{dP} = 0. \quad \text{(33)}
\]

From the results of Property 2, equation (32) has three roots with \(P_{11}^*, P_{12}^*, \text{ and } P_{13}^*\), and equation (33) has two roots with \(P_{21}^*\) and \(P_{22}^*\). Although \(\Pi_2(P)\) can be differentiated, the resulting equation is mathematically intractable; that is, it is impossible to find \(P_i^*\) in explicit form. Thus, we can find \(P_i^*\) by numerical search method. Consequently, only the elements in set \(\Omega = \{ P_{21}^*, P_{11}^*, P_{12}^*, P_{0}, \frac{a}{b} - \epsilon \}, \text{ where } \epsilon \text{ is very small positive number} \) become candidates for \(P^*\) because \(\Pi_2(P) = \Pi_2(P)\) at \(P = P_0\).

For \(P_{ij}^*, j = 1, 3\), to be a candidate of \(P^*\) in Case 1, each \(P_{ij}^*\) must lie on \([P_0, a/b]\) and also for \(P_{21}^*\) to be a candidate of \(P^*\) in Case 2, \(P_{21}^*\) must lie on \((0, P_0)\). For \(P_0\) to be a candidate of \(P^*\) in Case 1, \(\Pi_0(P)\) must be decreasing at \(P = P_0\). Also, for \(a/b - \epsilon\) to be a candidate of \(P^*\) in Case 1, \(\Pi_0(P)\) must be increasing at \(P = a/b - \epsilon\). Note that depending on the relative size of \(\hat{P}_{11}, \hat{P}_{12}, \hat{P}_{2}, \hat{P}_{0}\) and \(\frac{a}{b}\), each element in \(\Omega\) can be a candidate value or not. And therefore, some elements in \(\Omega\) can be dropped from consideration in search of \(P^*\).

Based on the above observations for the candidates of \(P^*\), we develop the following solution algorithm to determine the buyer’s optimal selling price and lot-size.
**Solution Algorithm**

**Step 1.** This step computes $\Pi_2^\delta(P)$ for the candidate values in set $\Omega$.

1.1. Compute $\Pi_2^\delta(P)$ at $P=P_0$ by equation (26).

1.2. If $\Pi_2^\delta(P_0)<0$, then compute $\Pi_2^\delta(P)$ at $P=P_{21}^*$ and go to Step 1.4.

   Otherwise, go to Step 1.3.

1.3. If $\hat{P}_2<P_0$, then compute $\Pi_2^\delta(P)$ at $P=P_{21}^*$ and go to Step 1.4.

   Otherwise, go to Step 1.4.

1.4. Determine $Q_2(P)$ by equation (8) and go to Step 2.

**Step 2.** This step computes $\Pi_1^\delta(P)$ for the candidate values in set $\Omega$.

2.1. Compute $\Pi_1^\delta(P)$ at $P=P_0$ by equation (17).

2.2. If $\Pi_1^\delta(P_0)\geq0$, then go to Step 2.3.

   Otherwise, go to Step 2.4.

2.3. Compute $\Pi_1^\delta(P)$ at $P=\frac{a}{b}+\epsilon$ by equation (17).

   2.3.1. If $\Pi_1^\delta(\frac{a}{b}+\epsilon)\geq0$, then go to Step 2.3.2.

   Otherwise, go to Step 2.3.4.

   2.3.2. If $\frac{a}{b}<\hat{P}_{11}$, then compute $\Pi_1^\delta(P)$ at $P=\frac{a}{b}+\epsilon$ and go to Step 2.5.

   Otherwise, go to Step 2.3.3.

   2.3.3. If $P_0<\hat{P}_{11}$, then compute $\Pi_1^\delta(P)$ at $P=P_{11}$, $\frac{a}{b}+\epsilon$ and go to Step 2.5.

   Otherwise, compute $\Pi_1^\delta(P)$ at $P=\frac{a}{b}+\epsilon$ and go to Step 2.5.

   2.3.4. If $\frac{a}{b}<\hat{P}_{12}$, then compute $\Pi_1^\delta(P)$ at $P=P_{11}$ and go to Step 2.5.

   Otherwise, go to Step 2.3.5.

   2.3.5. If $\frac{a}{b}<\hat{P}_{13}$, then compute $\Pi_1^\delta(P)$ at $P=P_{11}$ and go to Step 2.5.

   Otherwise, compute $\Pi_1^\delta(P)$ at $P=P_{11}, P_{13}$ and go to Step 2.5.

2.4. Compute $\Pi_1^\delta(P)$ at $P=\frac{a}{b}+\epsilon$.

   2.4.1. If $\Pi_1^\delta(\frac{a}{b}+\epsilon)\geq0$, then compute $\Pi_1^\delta(P)$ at $P=P_0$, $\frac{a}{b}+\epsilon$ and go to Step 2.5.

   Otherwise, go to Step 2.4.2.

   2.4.2. If $\hat{P}_{12}<P_0$, then compute $\Pi_1^\delta(P)$ at $P=P_0$ and go to Step 2.5.
Otherwise, go to Step 2.4.3.

2.4.3. If \( \frac{a}{b} < \hat{P}_{12} \), then compute \( \Pi_1^0(P) \) at \( P = P_0 \) and go to Step 2.5.

Otherwise, compute \( \Pi_1^0(P) \) at \( P = P_0, P_1^* \) and go to Step 2.5.

2.5. Determine \( Q_1(P) \) by equation (7) and go to Step 3.

Step 3. Select the optimal selling price(\( P^* \)) and lot-size(\( Q^* \)) which gives the maximum annual net profit among those obtained in steps 1 and 2.

4. Numerical Example

As a marketing policy, some suppliers offer credit periods to the buyers for the stimulation of product demand. Trade credit is an effective means of reducing the cost of holding stocks. Therefore, the availability of the credit period from the supplier enables the buyer to discount the selling price, expecting to increase the customer’s demand. In this regard, we considered the situation that the customer’s demand rate is a downward slopping linear function of the buyer’s selling price. The following example will be used to illustrate the solution algorithm.

Let the customer’s demand rate be the linear function and let \( \hat{a} = 10.000 \) and \( \hat{b} = 1.250 \). That is, \( D = 10.000 - 1.250 \cdot P \). Also, let \( C = \$3, S = \$50, t_c = 0.3, H = \$0.1, R = 0.15(=15\%) \) and \( I = 0.1(=10\%) \). In order to solve the problem, a computer program written QBASIC was developed to find an optimal solution as follows;

Step 1.1. From equation (14), \( P_0 = 5.78 \). Then, compute \( \Pi_1^0(P) \) at \( P = 5.78 \) by equation (26).

Step 1.2. Since \( \Pi_1^0(P_0) = -790 < 0 \), compute \( \Pi_1^0(P) \) at \( P = P_{21}^* \).

Solving equation (18) numerically on the price interval, \( P < P_0 \), we obtain \( P_{21}^*(=5.46) \) and compute \( \Pi_1^0(P) \) at \( P = 5.46 \).

Step 1.4. Compute \( Q_2(P) \) at \( P = 5.46 \) by equation (8) and go to Step 2.

Step 2.1. Compute \( \Pi_2^0(P) \) at \( P = 5.78 \) by equation (17).

Step 2.2. Since \( \Pi_2^0(P_0) = -732 < 0 \), go to Step 2.4.

Step 2.4. Compute \( \Pi_2^0(P) \) at \( P = \frac{a}{b} - \epsilon (= 7.99) \) by equation (17).

Step 2.4.1. Since \( \Pi_2^0(7.99) = 35040 > 0 \), compute \( \Pi_2^0(P) \) at \( P = 5.78, 7.99 \) and go to Step 2.5.

Step 2.5. Compute \( Q_1(P) \) at \( P = 5.78, 7.99 \) by equation (7) and go to Step 3.

Step 3. Since \( \Pi_2^0(5.46) = 7739.88, \Pi_2^0(5.78) = 2178.84 \) and \( \Pi_2^0(7.99) = 37.82 \), an optimal solution \( (P^*, Q^*) \) becomes \( (5.46, 891) \) with its maximum annual net profit $7739.88.
To determine the buyer’s optimal selling price and lot-size, the availability of the credit period from the supplier enables the buyer to choose the selling price from a wider range of price option ($a/b = 8$). Following the solution algorithm, we only need to consider $P = 5.46, 5.78, 7.99$ for finding the buyer’s optimal value. After comparing the profit values with $P = 5.46, 5.78, 7.99$, the buyer’s optimal selling price ($P^*$) becomes 5.46 and lot-size ($Q^*$) becomes 891 with its maximum annual net profit $\$7739.88$.

5. Conclusions

This paper dealt with the buyer’s joint price and lot-size determination problem when the supplier offers a certain fixed credit period. Recognizing that the major reason for the supplier to offer a credit period to the buyers (retailers) is to boost up the demand of the product, we expressed the customer’s demand rate of the product with the downward slopping linear function. The function value depends on the buyer’s selling price which in turn is affected by the length of credit period. After formulating the mathematical model, we found the characteristics of the buyer’s annual net profit function. As proved in appendices A and B, we showed the shape of the annual net profit and analyzed how a buyer can determine the optimal selling price and lot-size. Although the model has a very complicated structure, we can find the optimal solution easily by the numerical search method. The results revealed that the availability of the credit period from the supplier enables the buyer to choose the selling price from a wider range of price option expecting an additional customer’s demand.

There are several interesting opportunities for future research in this subject. The model can be easily extended to the case of deteriorating product. While this paper focuses on a fixed delay in payments, the case of multi-level of trade credit could be investigated.

References


